Application of Grey System Theory to tree growth prediction

Wang Jing (王晶)*
Forestry Survey and Design Institute of Heilongjiang Province, Harbin 150080, P. R.China
Hou Yuesong(侯月松) Li Weilin（李伟林）
Shuangyashan Forestry Bureau of Heilongjiang Province, Shuangyashan 155100, P. R. China
Cheng Wenhui(成文惠)
Power University of Heilongjiang Province, Harbin 150030, P. R. China

Abstract Based on Grey System theory, tree growth prediction models are developed by using 202 temporary plots and 206 stem analysis trees of Dahurian larch (*Larix gemlinii* Rupr.) in 10 forestry bureaus of Yakeshi Forestry Administrative Bureau in Daxing'an Mountains of the Inner Mongolia Autonomous Region. By residual and posterior tests, their precisions are qualified. With several data, tree growth can be predicted using Grey System models. For DBH and volume, the fitting results of Grey System models are better than that of statistical models.
Key words: The Grey System, Tree growth prediction, Dahurian larch Plantations

Introduction

Dahurian larch plantations were planted after the 1950's in Heilongjiang Province. Some of them are less than 40 years old. There lack permanent plots of long-term. Sometimes, statistical models for predicting tree growth are not always desirable. The Grey System is a new branch of mathematics founded by Prof. Deng Julong in 1982 (Deng 1985). In contrast to statistic models, which need a lot of measured data, the Grey System only need a few data for setting up models. In recent years it has been successfully used in engineering control (Deng 1985), economic management (Deng 1986), social systems (Deng 1985), ecosystems (Ni 1987), tree diameter distribution (Ma 1999) and forecasting forest fire disaster area (Zhang 1999). On this study, we will apply the Grey System to the prediction of the tree growth for Dahurian larch plantations.

Data collection

Data for this study were collected from 202 temporary plots and 206 stem analysis trees located in Forestry Bureaus of Yakeshi Forestry Administrative Bureau in Daxing'an Mountains. Statistics for stand variables are presented in Table 1.

Tables 1. Stand Attributes of 202 Plots

Stand variable	Minimum	Maximum	Average	Standard error
Stand age (A), a.	6	42	19.134	5.610
Average DBH (Dg), cm	1.5	24.5	9.882	3.359
Mean height --H, m	2.4	14.7	8.852	2.699
Dominant height (Hd), m	3.2	16.3	10.459	2.893
Site index (SI), m	10.0	18.0	13.372	1.783
Number of trees(N), N/hm^2	250.0	10.667	2898.005	1520.152
Stand volume (m^3/hm^2)	1.0	247.0	99.277	49.939
Basal area (m^2/hm^2)	0.355	47.780	20.784	9.493

Methods and results

Assuming the time series of raw data

$$\{x^{(0)}(t)\} = \{x^{(0)}(1), x^{(0)}(2), \cdots, x^{(0)}(N)\}$$

is a random process (Deng 1987).

If we make AGO (accumulated generating operation), that is

$$x^{(1)}(t) = \sum_{t=1}^{k} x^{(0)}(k), \quad K=1, 2.....N$$

We attain a new time series:

*Wang Jing, male, born in April 1963, engineer, Forestry Survey and Design Institute of Heilongjiang Province.
Received: 1999-09-08
Responsible editor: Chai Ruihai

$$\{x^{(1)}(t)\} = \{x^{(1)}(1), x^{(1)}(2), \cdots, x^{(1)}(N)\} =$$

$$\{x^{(0)}(1), \sum x^{(0)}(2), \cdots, \sum x^{(0)}(t)\}$$

If the randomness is not weakened enough by one time's AGO, we can make it in times AGO. Then,

$$x^{(m)}(t) = \sum_{k=0}^{t} x^{(m-1)}(t) \qquad K=1, 2\ldots\ldots, N$$

The form of GM (1,1) is $\frac{dx^{(1)}(t)}{dt} + ax^{(1)}(t) = b$

Solving it

$$x^{(1)}(t) = \sum_{t=1}^{k} x^{(0)}(t) \qquad K=1, 2, 3\ldots\ldots, N$$

$$\hat{x}(t+1) = \left(x^{(0)}(t) - \frac{b}{a}\right)e^{-at} + \frac{b}{a}$$

Where: *a, b* is parameters, *t* is time, *x* is variable $X^{(0)}(1)$ is the first data of original data series, $X^{(1)}(t+1)$ is one time AGO data at the time of t+1.

By computing, tree growth prediction models can be developed (see Table 2).

Precision tests

Two kinds of precision tests are carried out:

Residual test

$$q(t) = \hat{X}^{(0)}(t) - x^{(0)}(t)$$

$$e(t) = \left(\hat{X}^{(0)}(t) - \hat{x}^{(0)} / x^{(0)}(t) \times 100\%\right)$$

Where, *q* is error, e is the percentage of the error

$\hat{X}^{(0)}(t)$ is the data restored from GM(1,1),

$X^{(0)}(t)$ is original data

Posterior residual tests

1) Computing the ratio posterior residual (*C*)

$$C = \frac{S_2}{S_1} = \frac{\sqrt{S_2^2}}{\sqrt{S_1^2}}$$

Where:

S_2^2=the square sum of residual deviation,

S_1^2= the square sum of deviation of X.

2) Computing the frequency of tests error

$P=P\{q(t)<0.6745\ S\}$

3) Deciding the grade of GM(1,1)

If $P>0.95$ and $C<0.35$, the grade of GM (1,1) is A (good).

The results of residual test are listed into Table 3.

Table 2. Tree growth prediction model*

Plot No	Factor	Growth prediction model
C21	DBH	*X*(t+1)=47.92479*exp(0.1305245*t)-44.52748
	H	*X*(t+1)=49.53118*exp(0.1319027*t)-44.43118
	V	*X*(t+1)=0.02961514*exp(0.3796691*t)-0.0269
C34	DBH	*X*(t+1)=36.19929*exp(0.1607297*t)-31.99929
	H	*X*(t+1)=31.47915*exp(0.1827793*t)-26.67915
	V	*X*(t+1)=0.0251069*exp(0.4138023*t)-0.02116
C35	DBH	*X*(t+1)=40.12162*exp(0.143068*t)-36.32162
	H	*X*(t+1)=45.00785*exp(0.1427506*t)-40.40785
	V	X(t+1)=0.1511248*exp(0.4500573*t)-0.14267
C36	DBH	*X*(t+1)=47.00009*exp(0.1340826*t)-42.30009
	H	*X*(t+1)=51.41670*exp(0.1258408*t)-45.81607
	V	*X*(t+1)=0.02696725*exp(0.4055373*t)-0.02227
C37	DBH	*X*(t+1)=38.24884*exp(0.1375806*t)-34.84884
	H	*X*(t+1)=48.84147*exp(0.135558*t)-43.741747
	V	*X*(t+1)=0.02252165*exp(0.3740589*t)-0.01997
A02	DBH	*X*(t+1)=35.28238*exp(0.19654688*t)-30.78238
	H	*X*(t+1)=55.45214*exp(0.1387165*t)-49.15124
	V	*X*(t+1)=0.04727944*exp(0.3797047*t)-0.04268
A04	DBH	*X*(t+1)=31.22261*exp(0.1698283*t)-29.66263
	H	*X*(t+1)=54.92002*exp(0.137597*t)-50.1020
	V	*X*(t+1)=0.01674117*exp(0.496139*t)-0.0106238
A44	DBH	*X*(t+1)=45.14847*exp(0.1500642*t)-41.4485
	H	*X*(t+1)=49.04505*exp(0.1503172*t)-42.945502
	V	*X*(t+1)=0.0400904*exp(0.3889082*t)-0.03677
A45	DBH	*X*(t+1)=52.05569*exp(0.1366392*t)-47.1133
	H	X(t+1)=51.88254*exp(0.1514621*t)-45.558252
	V	X(t+1)=0.03960343*exp(0.4274031*t)-0.03112
A47	DBH	*X*(t+1)=75.208769*exp(0.0925099*t)-70.52748
	H	*X*(t+1)=84.33929*exp(0.0971868*t)-77.73343
	V	*X*(t+1)=0.05121108*exp(0.291177*t)-0.04623

*Here only 10 plots models are listed .

Comparison of Statistical Models and Grey System Models

According to the best statistical models of age and other growth factors, the fitting results of No. 154 stem analysis tree in plot A02 are as flowing:

D=-1.679996+0.7333328A-1.847499E-08A^2

$H=25.69431(1-\exp(-0.031441A))^{1.001978}$

$V=0.000051797A^{2.365297}$

Where: A is age (a), D is DBH (cm), H is height (m), V is volume (m^3)

The error comparison was made using 10 plots. Here only the result of No. 154 stem analysis tree was presented in Table 4.

Table 3. Results of the residual tests *

Plot No.	DBH			Height			Volume		
	C	*P*	grade	*C*	*P*	grade	*C*	*P*	grade
C 21	0.0587	1	A	0.0179	1	A	0.0811	1	A
C 34	0.0823	1	A	0.0684	1	A	0.0913	1	A
C 35	0.1246	1	A	0.0795	1	A	0.1563	1	A
C 36	0.0563	1	A	0.0216	1	A	0.1093	1	A
C 37	0.0743	1	A	0.0440	1	A	0.1257	1	A
A 02	0.0649	1	A	0.0307	1	A	0.0734	1	A
A 04	0.0976	1	A	0.0185	1	A	0.0820	1	A
A 44	0.0545	1	A	0.0600	1	A	0.0868	1	A
A 45	0.0785	1	A	0.0301	1	A	0.0929	1	A
A 47	0.0414	1	A	0.0507	1	A	0.1049	1	A

*Here only 10 plots' residual tests are listed

Table 4 The error comparison of statistical models and grey system models

Age class	Growth factors	Statistical Models	Grey system models
12	DBH*	-0.88	-0.38
	Height	-0.04	0.15
	Volume*	-0.0012	-0.0019
15	DBH*	-0.12	0.06
	Height	-0.04	0.12
	Volume*	-0.0004	0.0001
18	DBH*	0.72	0.46
	Height	-0.01	-0.21
	Volume	-0.0023	-0.0040
21	DBH*	-0.38	0.30
	Height	-0.01	0.11
	Volume*	-0.0016	0.0004

*The result of Grey System models is better than that of statistical models.

From Table 4, we see that for DBH and volume, the fitting result of Grey System models is better than that of statistical models. But for the height it's opposite.

Conclusions

Using the theory of Grey System in Dahurian larch plantation growth prediction is feasible. We can use a small set of data from stem analysis to set up the tree growth prediction model. The precision is qualified. All of the grades of GM (1,1) are A (good). For DBH and volume, the fitting result of Grey System models is better than that of statistical models.

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